Model of structured continuum and relation between specific surface, porosity and permeability

Boris B. P. Sibiryakov, Lourenildo W. B. Leite and Wildney W. S.Vieira, UFPA, Brazil

Abstract

Contrary to the Cauchy and Poisson classical continuum, the new theory for structured or blocked media must contain several degrees of freedom. It is evident, because elementary blocks may transfer the motion by contact interactions, by rotations, and by group of particles. It means, that the energy contents is not only in the first derivatives (strains). The potential energy contents is in the second derivatives (curvatures) and in other higher order derivatives. It means that the equation of motion of blocked media should contain several order derivatives; in other words, the equation of motion may have very high, or, probably, an infinite derivative order.

Introduction

The present paper is part of the project submitted to Science Without Borders with the title *"The Prediction of Stresses and Strains using P and S Wave Velocities in Order to Localize Areas of Small Pressure in Oil and Gas Productive Layers as Natural Sucking Pumps"*. This project is devide in different relative independent steps.

In the first step we need to do conventional seismic investigations in order to obtain information about P and S wave velocities, and also about the configuration of seismic boundaries.

The second step is the prediction of stresses in the geological structures using the information obtained in the first step. Also, the prediction of nontrivial behavior of pressure, since it may decrease with depth, and creates natural pumps which accumulates fluids. We must predict these natural pumps.

The third step is the prediction of rupture in pressure between solid and fluid, that depend on the structure of pore space.

The present investigation is part of the third step described above.

In order to start to predict stress and strain for real geological structures, we need to know P and S velocites, and the configuration of seismic boundaries. This is a classical problem of seismic investigations. Also, the present description is restricted to isotropic models, and for anysotropic situations the equations are more complicated.

It is very important that the acquired data be three component. From land observed data, we can use S waves from horizontal vibroseis, together with VSP technology. From marine observed data, we can use AVO technology looking for converted P-S-P waves. In special cases, we can use petrophysical measurments of borehole data.

Equation of motion for structured media

The new model of structured continuum contains internal geometry of micro-inhomogeneous medium described mainly by porosity and specific surface.

The porosity, f, is described as a fraction of the empty space, V_E, to the total volume, V_T, of the material including the solid and empty space: $f = \frac{v_E}{v_T}$. The empty space may contain gas and liquid.

The specific surface area (SSA) is a relation between the real surface of pores and cracks to the volume of the specimen, $\sigma_0 = \frac{S}{V_T}$[cm^{-1}], it is used for solving petrophysic and chemical problems, and it is measured by Mercury (Hg), and by gas absorption methods. Examples of geometrical figures, for instance, from a tetrahedron to a sphere gives decaying values in the form $\sigma_0 \propto \frac{1}{a}$, where a is the solid parameter, like radius or side. (Mavko et al., 1999).

Figure 1 shows an element volume of a structured body, where l_0 is the average distance between pores. There is a theorem of Integral Geometry, which relates of the Specific Surface σ_0 to l_0 by the formula, Sibiryakov (2002),

$$\sigma_0 l_0 = 4(1-f). \qquad (1)$$

Figure 2 represents another situation, where we have the structure of the cracked media characterized by the specific surface, σ_0, or the arithmetic average distance, l_0, between cracks. In the grain media we have negative curvature of grains, but positive curvature for the pore space. In cracked media, we have as a rule zero curvature for the boundary porous/solid. Positive curvature of solid for cavernous pores gives us large pressure jump between solid and liquid. (Landau and Lifschitz, 1961)

In equation (1) f is the porosity; therefore, if there is a sample with specific surface σ_0, there is automatically an average size l_0 of the micro-structure. The distinction between classical and structured continuum should be clear in Figure 1. In the volume bounded by surface C there is an equation of equilibrium because all forces cancel each other, while in the volume bounded by surface D there is no equation of equilibrium because all forces concentrate on one side of the grain, and the other side does not have forces. The idea is to create a new space model for structured media.

We consider some finite body volume, where surface forces

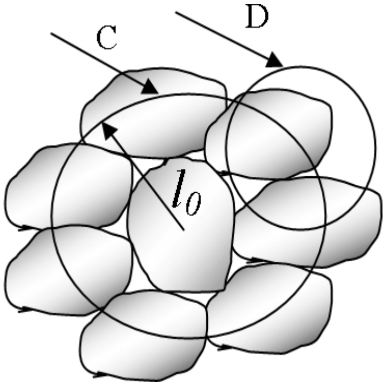

Figure 1: Element of structured body of granular medium, where l_0 is an average distance between cracks, or grains. The problem is to create an equation of equilibrium for arbitrary element of the discrete medium. For the surface C there is an equation of equilibrium, but on the surface D there is not.

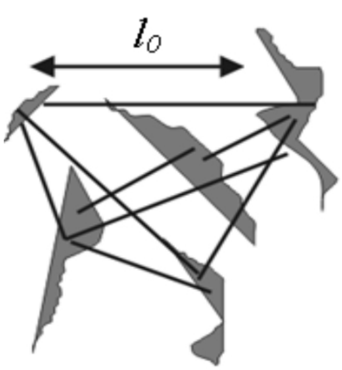

Figure 2: Another complex element of structured body of cracked medium, where l_0 is an average distance between cracks.

are applied on a sphere of radius l_0, while the inertial forces are applied in the structure center. There is no possibility to make an elementary volume tend to zero, coinciding to points on the surface, and the inertial forces like in classical continuum; we must consider a finite volume, as a representative body volume, and we have a problem of inertial forces at different positions on the surface.

The main feature of this approach is to fill all space, including pores and cracks, by force field. Due to such operation, we have continuous image of real complicate medium. The natural laws must apply to continuous image of the medium, and not to the real one. The one-dimensional operator for field translation from point x to point $x \pm l_0$ is given by the symbolic formula, Maslov (1973),

$$u(x \pm l_0) = u(x)e^{\pm l_0 D_x}. \tag{2}$$

This form applies to any field, but here $u(x,y,z,t)$ stands for displacement.

In formula (2), the field translation operator $D_x = \frac{\partial}{\partial x}$, for 3D space, can be rewritten as follows, Sibiryakov and Prilous (2007),

$$P(D_x,D_y,D_z) = \frac{\sinh(l_0\sqrt{\Delta})}{l_0\sqrt{\Delta}} = E + \frac{l_0^2\Delta}{3!} + \frac{l_0^4\Delta\Delta}{5!} + \ldots \tag{3}$$

where E is the unit operator, Δ is the Laplace operator, $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = \Delta$, and P is the special symbolic averaging operator given by:

$$P(D_x,D_y,D_z) = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} e^{l_0(D_x\sin\theta\cos\phi + D_y\sin\theta\sin\phi + D_z\cos\theta)}\sin\theta d\theta d\phi \tag{4}$$

In classical continuum, we apply the impulse conservation law, $F_i = m\ddot{u}_i$ to any element of the medium, or $F_i = \frac{\partial\sigma_{ik}}{\partial x_k} = \rho\frac{\partial^2}{\partial t^2}u_i$, where m stands for mass, and ρ for density.

In the present case, we need to fill all pore space by a force field $P(\frac{\partial\sigma_{ik}}{\partial x_k}) = \rho\frac{\partial^2}{\partial t^2}u_i$. Instead of real stresses, which may change from a large value (in the grain) down to zero (in the pore space), we use the image of real stresses. Namely, we use a continuous field constructed by application of the P operator to the real complicate force field. For this continuous image of real stresses, $P(\sigma_{ik})$, we can apply the impulse conservation law. In classical continuum model, this operation is made by nature itself. This model of the continuum requires some mathematical operations in order to create the continuum medium.

In one-dimensional case, plane waves, stationary motion, $u(x,y,z,t) \to u(x,\omega = k_B v_B; l_0)$, we have a simpler equation in the form, Sibiryakov and Prilous (2007),

$$\left(E + \frac{l_0^2}{3!}\frac{\partial^2}{\partial x^2} + \frac{l_0^4}{5!}\frac{\partial^4}{\partial x^4} + \ldots\right)u_{xx} + k_B^2 u(x,k_B;l_0) = 0, \tag{5}$$

where $k_B = \omega/v_B$ stands for both P and S waves. Considering only one term, we write equation (5) as:

$$u_{xx} + k_B^2 u = 0. \tag{6}$$

Considering only two terms, we write equation (5) as:

$$u_{xx} + \frac{l_0^2}{3!}u_{xxxx} + k_B^2 u = 0. \tag{7}$$

We can look at a solution of the equation of motion (5) in the stationary form,

$$u(x,y,z,t) = U(x,y,z)e^{i\omega t}. \tag{8}$$

For one dimensional case, $U(x,y,z)$ has the form $U(x)$, and:

$$U(x) = A(k)e^{ikx} = A(\frac{\omega}{v})e^{i\frac{\omega}{v}x}. \tag{9}$$

We are not applying Fourier transform, but looking at discrete values of the temporal radial frequency (ω), and of the wavenumbers (k_x,k_y,k_z).

Condition for negative Poisson coefficient

Substituting representation (9) into equation (5), we obtain the dispersion equation for the unknown wavelength k,

$$\frac{\sin(kl_0)}{kl_0} = \left(\frac{k_B}{k}\right)^2, \tag{10}$$

where $k_B = \frac{\omega}{v_B}$ is the wavenumber of the usual both P or S wave. Equation (10) is a transcendental equation with respect to the unknown value of k. For the condition $l_0 \to 0$, then $k \to k_B$, and it means that for infinite small structures we have usual wave velocities. In the case that l_0 is not very small, then $k < k_B$, the dispersion velocity $v = \frac{\omega}{k}$ is greater than v_B, and the v_P and v_S velocities are decreasing due to structure.

Numerical examination of equation (10) shows that the P-wave velocity decreases more rapidly than S-wave velocity. This result means that the ratio $\gamma = \frac{v_S}{v_P}$ may be greater than $\frac{1}{\sqrt{2}}$. For classical continuum model, $\gamma = \sqrt{\frac{\mu}{\lambda+2\mu}}$, and if $\lambda = 0$, than $\gamma = \sqrt{\frac{1}{2}} \approx 0.705$, where (λ, μ) are the Lamè parameters. Now, if in measuring we have $\gamma > 0.705$, we must have $\lambda < 0$. As a result, the Poisson coefficient $\sigma = \frac{1}{2}\frac{\lambda}{\lambda+\mu}$ is negative, because $\lambda < 0$ and of small value, such that the denominator is positive. Experimental observations for this strange result was first published by Gregory (1976). The reason for negative Poisson coefficient is due to dispersion phenomenon in structured media. The real Poisson coefficient measured in statics, not by wave propagation, does not give such strange result.

In Figure 3 it is shown the relation between P and S wavenumbers, $\frac{k_B(\omega)}{k_0(\omega)}$, versus the ratio $\varepsilon = \frac{l_0}{\lambda_B}$. It is clear that the P wavenumber (curve 1) increases faster than the S wavenumber (curve 3), what means that the P-wave velocity decreases faster than the S-wave velocity. The ratio $\frac{v_S}{v_p}$ increases from 1 to 1.25, from low to high frequencies.

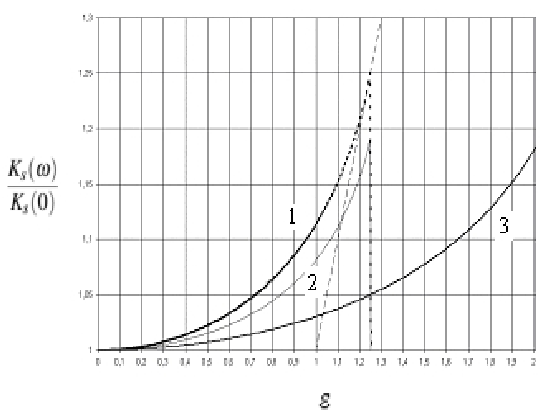

Figure 3: The horizontal axis gives $2\pi l_0/\lambda_B$ ratio. Curve 1 shows the increase of the wavenumber ratio $k_P(\omega)/k_0(\omega)$; i.e., decrease of v_P by increasing frequency. Curve 3 means the same for S waves. Curve 2 gives the increase of the v_S/v_P ratio up to negative Poisson coefficient σ.

Figure 4 shows the real and imaginary parts of the roots of the dispersion equation (10) as a function of $\varepsilon = \frac{l_0}{\lambda_B}$. The roots of equation (10) are obtained for $kl_0 = n\pi$, (n integer), and k very large, velocity very small. The interpretation of this figure is that if $\varepsilon \ll x$, then there is wave with abnormal small velocities less than v_S. Besides this conclusion, velocities are discrete in blocked media, while velocities are continuous in classic media. This situation is analogous in quantum mechanics (the discrete points represent discrete spectrum of eigenvalues).

No wonder, equation (10) contains derivatives of infinite order, and this circumstance is due to the several degrees of freedom for structured bodies. For $l_0 \to 0$, we have the usual equations of motion for classical continuum space model.

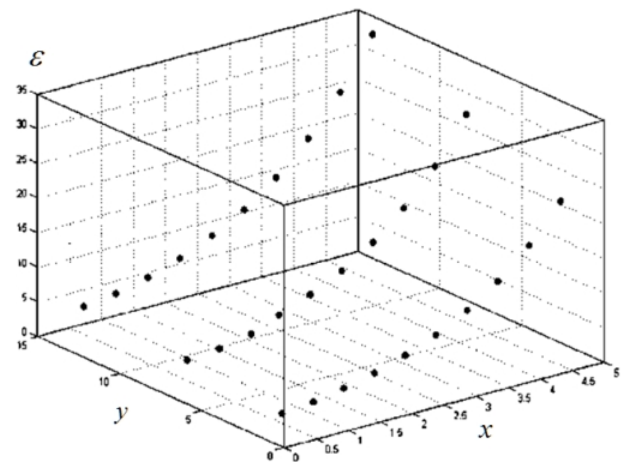

Figure 4: Position of complex roots of equation (10). The horizontal coordinate x corresponds to Real(kl_0), and y to Imag(kl_0). The vertical coordinate ε is $k_B l_0$. If $k_B l_0 \ll 1$, there are only real roots.

Blocked media with viscous liquid

Let us suppose that in the center of gravity (i.e., in the point x) the particle velocity is $\dot{u}_i(x)$. The average distance from point x to the boundary of grain is fl_0, and it means that the velocity on the grain boundary is represented by a sum of derivatives of the Taylor expansion in the form,

$$\dot{u}_i(x+fl_0) = \dot{u}_i(x) + fl_0\left(\frac{\partial \dot{u}_i}{\partial x}\cos(r,x) + \frac{\partial \dot{u}_i}{\partial x}\cos(r,y) + \frac{\partial \dot{u}_i}{\partial x}\cos(r,z)\right) + O((fl_0)^2). \quad (11)$$

This expansion is bounded to the first order term with respect to fl_0. This sum is equal to zero due to viscous sticking, and the derivative acts at the point x. We can suppose that the derivatives act on the contact skeleton-fluid with accuracy up to small values of the second order, and this means that,

$$\frac{\partial \dot{u}_i}{\partial r} = \frac{\partial \dot{u}_i}{\partial n}\cos(r,n). \quad (12)$$

The average value of $\cos(r,n)$ in three-dimensional space is 0.5, and that there is a relation between the derivative with respect to normal, $\frac{\partial \dot{u}_i}{\partial n}$, for the particle velocity and the velocity, $\dot{u}_i$, itself in the center, x, of the pore, namely,

$$-\dot{u}_i = \frac{1}{2} fl_0 \frac{\partial \dot{u}_i}{\partial n}. \quad (13)$$

The surface force, φ_i, of viscous friction is proportional to viscosity, η, and to the derivative of particle velocity, $\dot{u}$, with respect to the normal, n, to the surface that separates matrix and fluid,

$$\varphi_i = \eta \frac{\partial \dot{u}_i}{\partial n} = -2\frac{\eta}{fl_0}\dot{u}_i. \quad (14)$$

The volume force of viscous friction is a product of the surface force and of the specific surface area of the pore space, i.e.,

$$F_i = \sigma_0 \eta \frac{\partial \dot{u}_i}{\partial n} = -2\frac{\sigma_0 \eta}{f l_0}\dot{u}_i = 8\frac{\eta(1-f)}{f l_0^2}\dot{u}_i. \qquad (15)$$

In the case of the classical continuum model, the viscous friction gives the Telegraph equation (17) instead of the Wave equation (16):

$$u_{xx} = \frac{1}{c^2}\ddot{u}. \qquad (16)$$

In the structured continuum model, the forces created by internal stresses act by the P-operator.

In the formula (15), the factor $\frac{1}{2f}\sigma_0^2$ plays the role of inverse permeability. It means that the permeability ϕ is a geometric parameter, and it is equal $\phi = 2f/\sigma_0^2$ shown in Figure 5, from where we learn that σ_0 diminishes permeability very fast.

Such point of view shows that it is not necessary to use Darcy law for wave physics of percolation.

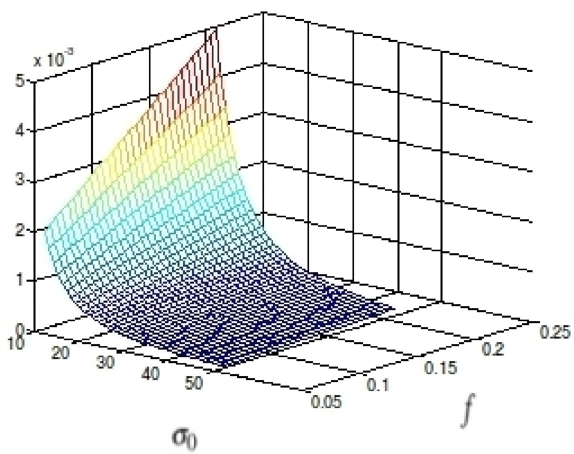

Figure 5: The horizontal axis are porosity f and specific surface σ_0, and the vertical axis is the permeability ϕ.

The equation (5) of motion is of infinite order, because in micro-inhomogeneous bodies there are many internal waves with different velocities. The Cauchy-Poisson hypothesis is that $P = E$, what means that any property is an average for any arbitrary small volume. In reality, we do not have the possibility to use arbitrary small volumes, because the representative volume must contain some elementary structures (grains), and they have internal motions, rotations and so on. If $l_0 \Rightarrow 0$, we have the classical standard equation of motion, and $P = E$.

For usual classical continuum ($P = E$), there is a simpler equation in the form,

$$u_{xx} = \frac{1}{c^2}\ddot{u} + \frac{\beta}{f}\dot{u}, \qquad (17)$$

where $\beta = \sigma_0^2 \frac{\eta}{\lambda+2\mu}$ $[TL^{-2}]$ is a characteristic of stress relaxation time for P waves, and analogous formula, $\beta = \sigma_0^2 \frac{\eta}{\mu}$, for S waves.

The solution of equation(17) for stationary vibration in the case of small parameter values, $\beta/f \Rightarrow 0$, gives an attenuation with constant decrement, which does not depend on frequency. For large parameter values, $\beta/f \Rightarrow \infty$, there is a solution with an attenuation that is proportional to the square root of frequency, $\propto \sqrt{\omega}$.

For blocked media, the viscous friction creates velocities only by fluctuation of particles, and it means that the friction forces act by the $P-E$ operator (Sibiryakov et al., 2011). The equation of motion in this blocked and viscous media can be represented in the form,

$$P\left(\frac{\partial \sigma_{ik}}{\partial x_k}\right) - \frac{i\omega\beta}{2f}\sigma_0^2(P-E)U + k_B^2 U = 0, \qquad (18)$$

where $U = U(x,y,z,\omega)$. The correspondent dispersion equation is given by,

$$\frac{\sin(kl_0)}{kl_0}\left(k^2 - \frac{i\omega\beta}{2f}\sigma_0^2\right) + \frac{i\omega\beta}{2f}\sigma_0^2 = k_B^2. \qquad (19)$$

The case $l_0 \to 0$, $k^2 \to k_B^2$, means that in classical continuum media, with infinitely small structure size, the viscous friction is absent. For small values of l_0 there is approximately equality for wavenumbers,

$$k = k_B\left(1 + \frac{2}{3}i\frac{\omega\beta}{f}\right), \quad k_B = \frac{\omega}{v_{P,S}}. \qquad (20)$$

It means that the attenuation is proportional to the square of frequency with respect to the usual viscous liquid.

Results and Conclusions

At present, the fluid percolation theory based on Darcy's law means that we can ignore stress-strain state in solids. Besides that, percolation theory contains porosity, but do not contain specific surface, that creates forces to stop percolation. Instead of Darcy's law we need predict stress-strain in solid and the rupture of pressure between phases. This rupture depends on structure of pore space, and not on porosity only.

There is no necessity to use Darcy's law for determination of permeability, since it is a geometric property of porous medium. Permeability value is directly proportional to porosity, and inversely proportional to the square of the specific surface for a specimen.

The porosity and specific surface give a possibility to use alternative methods for measuring of permeability.

Equation of motion with long waves compared to the structure does result in the wave equation, but in the telegraph equation, that describes the propagation and diffusion of waves.

References

Gregory, A. R., 1976, Fluid saturation effect on dynamic elastic properties of sedimentary rocks: Geophysics, **41**, no. 5, 895–921.

Landau, L., and Lifschitz, M., 1961, Course of theorectical physics: theory of elasticity.: Nauk, Moscow.

Maslov, V. P., 1973, Theory of operators: Nauka, Moscow.

Mavko, G., Mukerji, T., and Dvorkin, J., 1999, The rock physics handbook: Cambridge University Press, London.

Sibiryakov, B. P., and Prilous, B. I., 2007, The unusual small wave velocities in structural bodies and instability of pore or cracked media by small vibration: WSEAS Transactions on Applied and Theoretical Mechanics Issue 7, **2**.

Sibiryakov, B. P., Prilous, B. I., and Kopeykin, A. V., 2011, The nature of instabilities in blocked media and seismological law of gutenberg-richter: WSEAS Transactions on Applied and Theoretical Mechanics Issue 2, **6**, 69–79.

Sibiryakov, B. P., 2002, Supersonic and intersonic cracking in rock-like material under remote stresses: Theoretical and Applied Fracture Mechanics, **38**, no. 3, 255–265.

Acknowledgments

The authors would like to thank the Project INCT-GP. And in special to the Project Science Without Borders of CNPq/CAPES of the Brazilian Federal Government that sponsors this research aiming at oil exploration.